Avg. Rate of Change & Even/Odd Functions

U1 L5B ~~Analyzing~~ Graphs of Functions Name: _____ HR: _____

Skills: Use the Vertical Line Test, find zeros, determine increasing/decreasing intervals, find average rate of change, and identify even/odd functions.

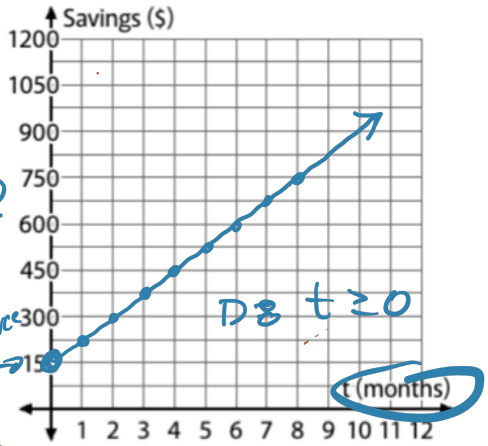
Slope: A measure of rate of change. $m = \frac{y_2 - y_1}{x_2 - x_1}$

Ex: Maya has $150 in her saving. When she started her after school job she began adding $75 a month to her saving.

Equation: t = time in months

$S(t) = 150 + 75t$

Graph:



$y = 75x + 150$ (slope)

$\frac{\Delta y}{\Delta x} = \frac{\$75}{1 \text{ month}}$

D: $t \geq 0$

Slope/Rate of Change

$m = \frac{\$75}{1 \text{ month}}$

Ex: Let f(x) = 150 + 75x Find the slope over the interval [1, 8]

$f(1) = 150 + 75(1) = \$225$ → (1, 225) → x_1, y_1

$f(8) = 750$ → (8, 750) → x_2, y_2

$m = \frac{750 - 225}{8 - 1} = \frac{525}{7} = 75$

The slope of a linear equation is 75

Ex: Find the slope of f(x) if $f(5) = -10$ **and** $f(-3) = 34$

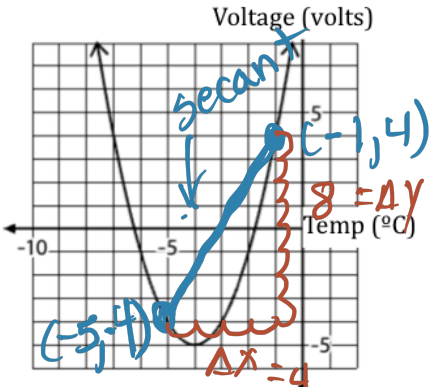
(5, −10) (−3, 34)

$m = \frac{34 - (-10)}{-3 - 5} = \frac{-11}{2} = \frac{\Delta y}{\Delta x}$

Average Rate of Change

This is just the slope formula applied to a curve — the slope of the secant line through two points.

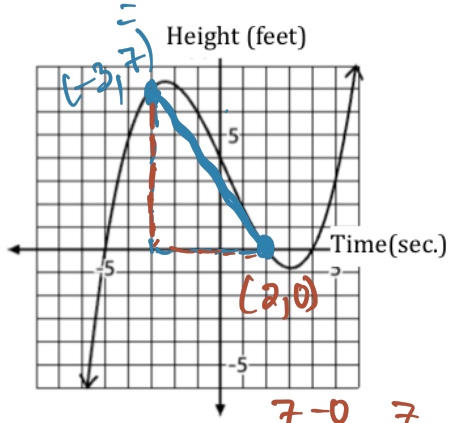
Ex. Find the average rate of change from $T = -5°C$ to $T = -1°C$



Secant

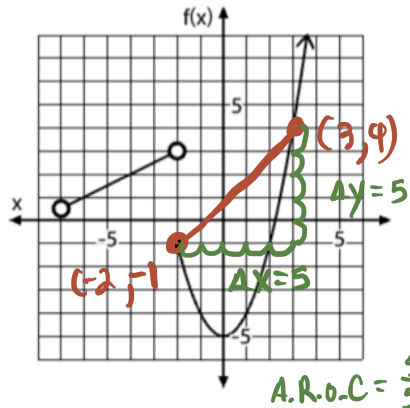
Avg. Rate of Change: $\frac{8}{4} = 2 \frac{\text{volts}}{°C}$

Ex: Find the average rate of change over the interval [-3, 2] seconds.



$m = \frac{7 - 0}{-3 - 2} = \frac{7}{-5} = \frac{-7}{5}$

Ex: Find the average rate of change over the interval [-2, 3]



A.R.O.C $= \frac{5}{5} = 1 = \frac{1}{1}$

$\text{Average Rate of Change} = m_{secant} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$ ($y_2 - y_1$)

Ex: f(x) = $x^3 - 3x$

(a) from $x_1 = -2$ to $x_2 = -1$:

$f(-1) = (-1)^3 - 3(-1) = -1 + 3 = 2$

$f(-2) = (-2)^3 - 3(-2) = -8 + 6 = -2$

Avg. A.R.O.C $= \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(-1) - f(-2)}{-1 - -2} = \frac{2 - -2}{-1 + 2} = \frac{4}{1} = 4$

(b) from $x_1 = 0$ to $x_2 = 1$:

$f(0) = (0)^3 - 3(0) = 0$

$f(1) = (1)^3 - 3(1) = 1 - 3 = -2$

A.R.O.C $= \frac{-2 - 0}{1 - 0} = \frac{-2}{1} = -2$

You try: f(x) = $x^2 + 2x$

(a) from $x_1 = -3$ to $x_2 = -2$:

$f(-3) = (-3)^2 + 2(-3) = 9 - 6 = 3$ → (−3, 3) x_1, y_1

$f(-2) = (-2)^2 + 2(-2) = 4 - 4 = 0$ → (−2, 0) x_2, y_2

A.R.O.C $= \frac{0 - 3}{-2 - -3} = \frac{-3}{-2 + 3} = \frac{-3}{1} = 3 = \frac{3}{1}$

(b) from $x_1 = -2$ to $x_2 = 0$:

$f(-2) = (-2)^2 + 2(-2) = 4 - 4 = 0$

A.R.O.C $= \frac{0 - 0}{0 - -2} = \frac{0}{2} = 0$

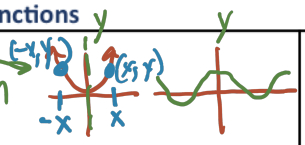
$f(0) = 0^2 + 2(0) = 0$



Even and Odd Functions

Even

f(−x) = f(x)



symmetric about the y-axis: Ex: $g(x) = x^4 + 7$

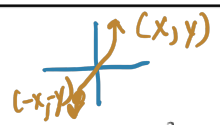
$g(-x) = (-x)^4 + 7$

$g(x) \quad x^4 + 7$

Even $\quad g(-x) = g(x)$

Odd

f(−x) = −f(x)



symmetric about the origin: $M(x) = x^3 - x$

$f(-x) = (-x)^3 - (-x)$

$= -x^3 + x$

$-f(x) = -(x^3 - x)$

$= -x^3 + x$

$f(-x) = -f(x)$ ODD

Neither

f(−x) matches neither f(x) nor −f(x)

Ex: $f(x) = 10 + 4x$

$f(-x) = 10 + 4(-x)$

$= 10 - 4x$ Not Even

$f(-x) \neq f(x)$

$-f(x) = -10 - 4x$ Not ODD

Ex: $g(x) = x^4 - x^2 - 1$

$g(-x) = (-x)^4 - (-x)^2 - 1$

$= x^4 - x^2 - 1$

Even

Find the Average Rate of Change over the interval: [-2, -1]

Ex: $h(x) = 2x^3 + 3x$

$h(-x) = 2(-x)^3 + 3(-x)$

$= -2x^3 - 3x$

$-h(x) = -(2x^3 + 3x)$

$= -2x^3 - 3x$

$h(-x) = -h(x)$ ← ODD

You try: $f(x) = x^3 - 4x$

$f(-x) = (-x)^3 - 4(-x)$

$= (-1 \cdot x)^3 - 4(-1 \cdot x)$

$(-1)^3(x)^3 + 4 \cdot x$

$-1 \cdot x^3 + 4x$

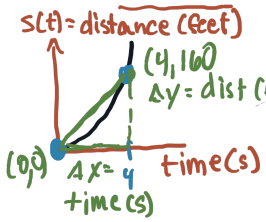
$-x^3 + 4x$

Application: Average Speed

A car's distance (ft) from a stoplight is $s(t) = 20t^{3/2}$, t in seconds.

$s(t)$ = distance, t = time

Find the average speed from t=0 to t=4 sec.



$s(0) = 20(0)^{3/2} = 0$

$s(4) = 20(4)^{3/2}$

$20(\sqrt[2]{4})^3$

$20(2)^3 = 160$

$\frac{\Delta y}{\Delta x} = \frac{\text{dist(ft)}}{\text{time(s)}} = \text{ft/s} = \frac{160 - 0}{4 - 0} = 40$ ft/s

You try: find the average speed from t=0 to t=9 sec.

Section 1.5 Summary — Can you...

- ☐ Use the Vertical Line Test?
- ☐ Find the zeros of a function?
- ☐ Identify increasing/decreasing intervals and relative extrema?
- ☐ Find the average rate of change between two points?
- ☐ Determine if a function is even, odd, or neither?

Formulas at a Glance — Section 1.5

A.R.o.C. (average rate of change) =

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Even: f(−x) = f(x)

Odd: f(−x) = −f(x)