U1 L4 Functions

Name: _____ HR: _____

Skills: Determine whether a relation is a function, use function notation, evaluate functions, find domains, and evaluate a difference quotient.

Review/Practice

1. Symmetry: check x, y, and origin

$y = |x^3 + 3x|$

x-axis: replace y with -y → $-y = x^3 + 3x$ → $y = -x^3 - 3x$ — No

y-axis: replace x with -x → $y = (-x)^3 + 3(-x)$ → $y = -x^3 - 3x$ — No

origin: $x \to -x$ & $y \to -y$ → $-y = (-x)^3 + 3(-x)$ → $-y = -x^3 - 3x$ → $y = x^3 + 3$ — Yes

2. Symmetry: check x, y, and origin

$y = \frac{3}{x^4 - 2}$

x-axis: $-y = \frac{3}{x^4-2}$ → $y = \frac{-3}{x^4-2}$ — No

y-axis: $y = \frac{3}{(-x)^4-2}$ → $y = \frac{3}{x^4-2}$ — Yes

origin: $-y = \frac{3}{(-x)^4-2}$ → $-y = \frac{3}{x^4-2}$ — No

3. Symmetry: check x, y, and origin

$y = 3|x + 2| - 3$

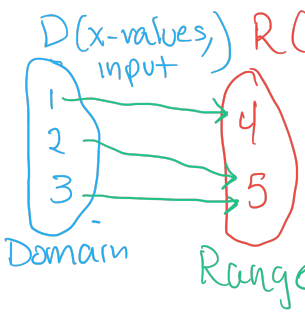
x-axis: $-y = 3|x+2| - 3$ → $y = -3|x+2| + 3$ — No

y-axis: $y = 3|-x+2| - 3$ → $y = 3|-1(x-2)| - 3$ → $y = 3|x-2| - 3$ — No

origin: $-y = 3|-x+2| - 3$ — No

• relation	any pairing of inputs (x) with outputs (y)
• function	a relation where each input has exactly one output
• domain	the set of all possible inputs (x-values)of a function
• range	the set of all possible outputs (y-values) of a function
• function notation	y = f(x); f is the function's name, x is the input, f(x) is the output

$\{(0,1),(2,3)\}$ (input, output)



What Makes a Relation a Function?

A function assigns to each input exactly one output. If any input repeats with a different output, it is NOT a function.

Ex: Is this a function? Testing function definition with points

Relation — function? No (2 → 11, 2 → 10)

Input, x	2	2	3	4	5
Output, y	11	10	8	5	1

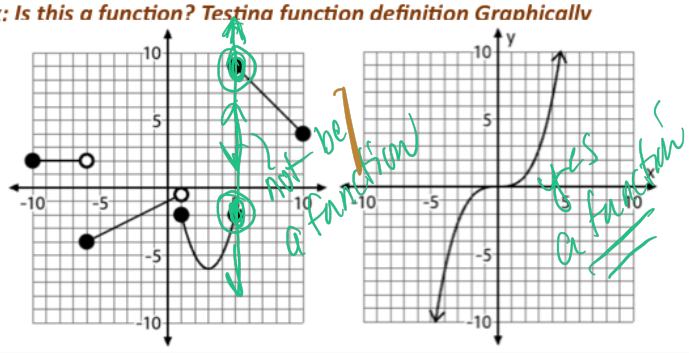
Ex: Is this a function? Testing function definition with equations.

Solve for y

a. $x^2 + y^2 = 8$ → $y^2 = 8 - x^2$ → $y = \pm\sqrt{8-x^2}$ — Not a function

b. $2y - 4x^2 = 36$ → $\frac{2y}{2} = \frac{4x^2}{2} + \frac{36}{2}$ → $y = 2x^2 + 18$ — function

Ex: Is this a function? Testing function definition Graphically



Not a function — Yes a function

Function Notation & Evaluating Functions → Pre-req SKill

Ex: f(x) = 3x – 5. Find f(1), f(–3), and f(x + 2).

$f(1) = 3(1) - 5 = 3 - 5 = -2$ → (1, -2)

$f(-3) = 3(-3) - 5 = -9 - 5 = -14$ → (-3, -14)

$f(x+2) = 3(x+2) - 5 = 3x + 6 - 5 = 3x + 1$ → (x+2, 3x+1)

You try: f(x) = x² + 1. Find f(2), f(–4), and f(t + 1).

$(x+h)(x+h)$

You try: f(x) = $g(x) = -x^2 + 4x + 2$. Find $f(x + h)$

$g(x+h) = -(x+h)^2 + 4(x+h) + 2$

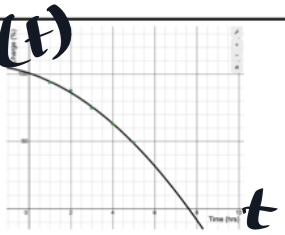
$-(x^2 + 2xh + h^2) + 4x + 4h + 2$

$-x^2 - 2xh - h^2 + 4x + 4h + 2$

Four Ways to Represent a Function

1. Verbally	2. Numerically	3. Graphically: Points on a coordinate plane.	4. Algebraically
A sentence describing how input relates to output.	A table or list of ordered pairs.		An equation in two variables.
Ex. Energy consumption on your phone as a function of time using the internet.			$E = -1.1t^2 - 5.1t + 100$
			$E(t) = -1.1t^2 - 5.1t + 100$

input t in hrs	1	2	3	4	5
output $E(t)$ in %	94	88	75	63	49



independent variable (x)

E(t)

Finding the Domain of a Function

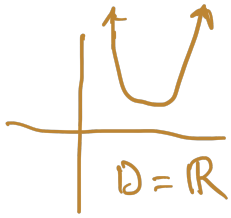
Unless stated otherwise, the domain is all real numbers EXCEPT values that make a denominator zero or a square root negative.

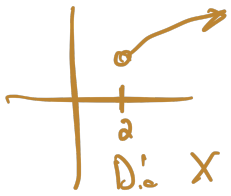
Domain: A description of the x values that can be used in an function or is used by the function.

Ex: $f(x) = x^2, x \geq 0$. (domain)

Ex: $f(x) = \begin{cases} x+2 & \text{if } x < 1 \\ 3x-1 & \text{if } x > 1 \end{cases}$

Graphically: The domain is all the x-values that are used to make the entire graph.







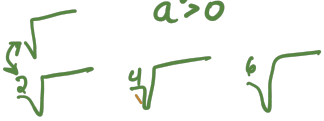
Why would any x-values NOT be allowed?

1. Physics/science/reality of the physical world: time, $t \geq 0$; mass, $m \geq 0$; age, $a \geq 0$

2. Math

a. You can't divide by 0

b. You can't take Even roots of negative numbers



c. You can't take the logarithm of 0 or negative numbers. Log (0) ← Not allowed

d. The Problem your trying to solve requires it.

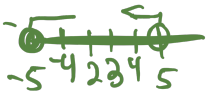
Counting People: D = Positive Integers + 0

Ex $f(x) = \sqrt{2x-9}$ ← Radicand

$2x - 9 \geq 0$

$\frac{2x}{2} \geq \frac{9}{2}$

$x \geq 4.5$



Domain: D: $x \geq 4.5$

Ex $f(x) = \sqrt{25 - x^2}$

$25 - x^2 \geq 0$

$25 \geq x^2$

$x^2 \leq 25$

$x \leq \pm 5$

$x \leq 5$ and $x \geq -5$

Domain:

Ex $f(x) = \frac{x-8}{x-3}$

$x - 3 \neq 0$

$x \neq 3$

Domain: D: ℝ, $x \neq 3$

$h(x) = \frac{x}{x^2 - x - 6}$

$x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$x \neq -2$ $x \neq 3$

Domain: D: ℝ, $x \neq -2, 3$

Finding Values for Which $f(x) = 0$

$f(x) = 4x - 15$, find all real values of x for which $f(x) = 0$

$4x - 15 = 0$

$4x = 15$

$x = 15/4 = 3.75$

$g(x) = x^2 - 5x + 6$, find all real values of x for which $g(x) = 0$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x = 2, x = 3$