U1 L5A Analyzing Graphs of Functions

Name: _____ HR: _____

Skills: Use the Vertical Line Test, find zeros, determine increasing/decreasing intervals, find average rate of change, and identify even/odd functions.

Warm-Up: Prerequisite Skills

You'll use these three skills constantly in this lesson — make sure they're solid before we start.

1. Evaluating f(−x)

Ex: f(x) = x² − 3x. Find f(−2).

$f(-2) = (-2)^2 - 3(-2)$

$= 4 + 6 = 10$

Ex: f(x) = 2x³ + x. Find f(−1):

$2(-1)^3 + -1$

$2(-1) - 1$

$-2 - 1 = -3$

2. Distributing a Negative

Ex: $-(x^3 - 4x) = -x^3 + 4x$

Ex: $-(2x^2 - 5x + 1) = -2x^2 + 5x - 1$

3. Factoring a Trinomial

Ex: factor x² − 5x + 6.

$(x-2)(x-3)$

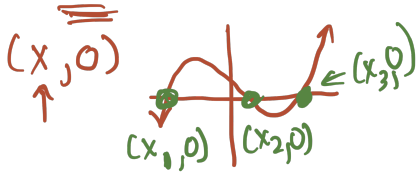
Ex: Factor x² − 7x + 10:

$(x-2)(x-5)$

Vocabulary

2/4

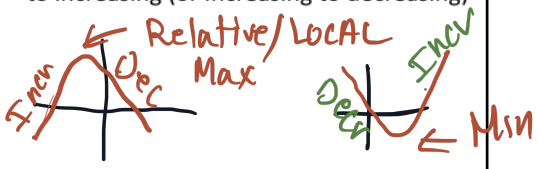
- Zeros of a function — the x-values for which f(x) = 0 (also the x-intercepts)



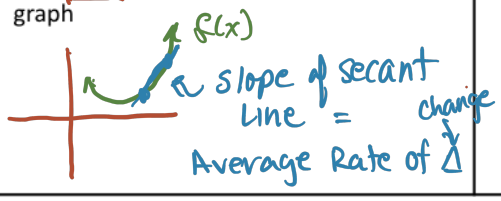
- Increasing / Decreasing / Constant — describes whether a graph rises, falls, or stays flat when read left to right



- Relative Minimum / Maximum — a point where the graph changes from decreasing to increasing (or increasing to decreasing)

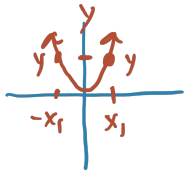


- Average Rate of Change — the slope of the secant line through two points on a graph



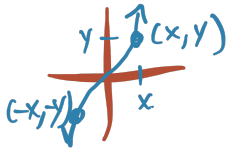
- Even function: If f(−x) = f(x) (symmetric about the y-axis)

$f(-x) = f(x)$



- Odd function: f(−x) = −f(x) (symmetric about the origin)

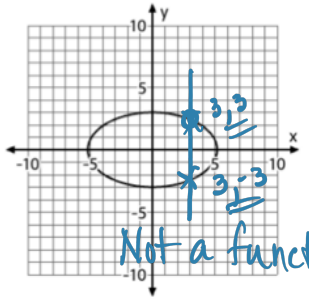
$f(-x) = -f(x)$

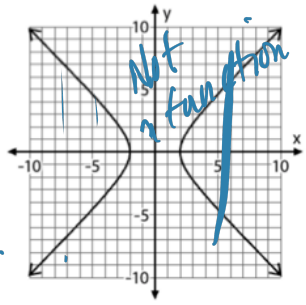


The Vertical Line Test & Zeros of a Function

Vertical Line Test

If any vertical line cross a graph more than once, the graph does NOT represent a function.





Ex: Find the zeros of f(x) = x² − 9x + 14. Using Desmos



$x^2 - 9x + 14 = 0$

$(x-2)(x-7)$

$x = 7, 2$

You try: Find the zeros of f(x) = 3x² − 16x + 21. Using TI-84

2.333333, 3

2 1/3, 3

7/3, 3

Determining Zeros Algebraically

Finding Zeros of functions Algebraically

a. $f(x) = x^2 - 9x + 14$.

$x^2 - 9x + 14 = 0$

$(x-2)(x-7) = 0$

$x = 2, x = 7$

b. $f(x) = \sqrt{15 - x^2}$ $x = \pm\sqrt{15}$

$(\sqrt{15-x^2})^2 = (0)^2$

$15 - x^2 = 0$

$-x^2 + 15 = 0$

$-15 \quad -15$

$-x^2 = -15$

$\sqrt{x^2} = \sqrt{15} \rightarrow x = \pm\sqrt{15}$

c. $f(x) = \dfrac{x^2 - 7x + 6}{x - 3}$ $x - 3 \rightarrow x \neq 3$

$x^2 - 7x + 6 = 0$

$(x-1)(x-6) = 0$

$x = 1 \quad x = 6$

Finding Zeros of functions Algebraically

a. $f(x) = \dfrac{(x-5)(x+4)^2}{x+1}$.

$(x-5)(x+4)^2 = 0$

$x-5=0$ $x=5$; $x+4=0$ $x=-4$; $x+4=0$

$x = 5, -4$

b. $f(x) = \sqrt{5x+12}$

$5x+12 = 0$

$5x = -12$

$x = -12/5$

$2x-10=0$

$2x=10$

$x=5$ ← $(2x-10)$

c. $f(x) = 4x^2 - 100$

$4x^2 - 100 = 0$

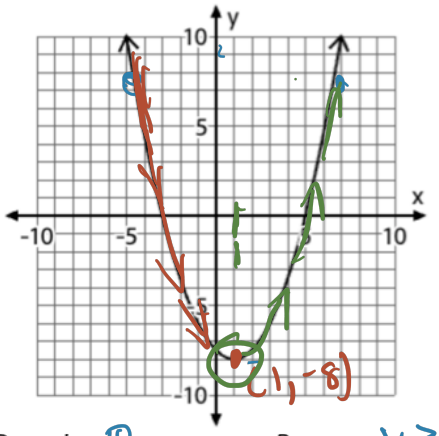
$\frac{4x^2}{4} = \frac{100}{4}$

$\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$

Increasing, Decreasing, and Relative Extrema

A relative minimum/maximum happens exactly where a graph switches from decreasing to increasing (or increasing to decreasing).

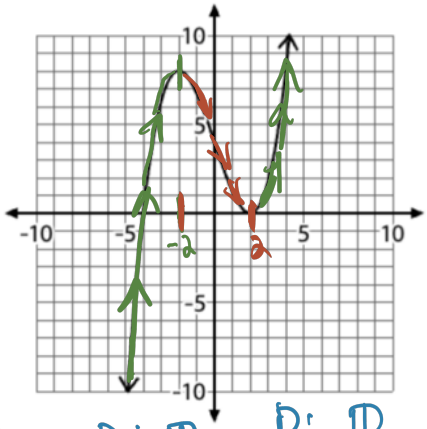


x *Domain:* $\mathbb{R}$ *Range:* $y \geq -8$

Increasing: $x > 1$ $(1, \infty)$

Decreasing: $x < 1$ $(-\infty, 1)$

Local min/max of -8 *at* $x = 1$ (y-value)

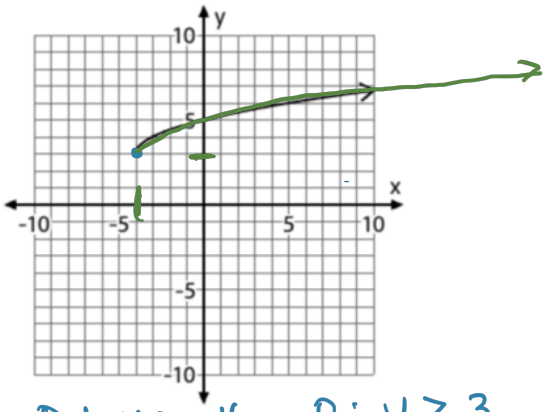


D: $\mathbb{R}$ R: $\mathbb{R}$

Increasing: $x < -2$ or $x > 2$ $(-\infty, -2) \cup (\infty, 2)$

Decreasing: $-2 < y < 2$ $(-2, 2)$

Local min/max: Local MAX: 8 at $x = 2$; min: 0 at $x = 2$



D: $x \geq -4$ R: $y \geq 3$

Increasing: $x > -4$ $(-4, \infty)$

Decreasing: ——

Local min/mas: LOCAL MIN 3

Definition-Range: ***The set of all output values of the function.***

Directions: *Determine the asked for quantities using a graphing calculator or demos.*

a. $f(x) = -x^2 - 4x + 3$.

Domain: $\mathbb{R}$ or $(-\infty, \infty)$

Range: $y \leq 7$ OR $(-\infty, 7)$

Increasing: $x < -2$ $(-\infty, -2)$

Decreasing: $x > -2$ or $(-2, \infty)$

Local min/max: $y = 7$

Zeros: $-4.65, .65$

Y-intercept: $(0, 3)$



b. $f(x) = -3\sqrt{6-x}$

$6 - x \geq 0$

$-x \geq -6$

$x \leq 6$

Domain: $x \leq 6$ $(-\infty, 6]$

Range: $y \leq 0$ $(-\infty, 0]$

Increasing: $(-\infty, 6)$

Decreasing: ——

Local min/max: $y = 0$

Zeros: $x = 6$

Y-intercept: -7.35

c. $f(x) = \dfrac{(x-4)(x+6)}{x-4}$ → $x - 4 = 0$, $x \neq 4$

$y = x + 6$

$y = 4 + 6 = 10$

Domain: $\mathbb{R}, x \neq 4$

Range: $\mathbb{R}, y \neq 10$

Increasing: $(-\infty, \infty), x \neq 4$

Decreasing: ——

Local min/max: ——

Zeros: $x = -6$

Y-intercept: $y = 6$