U1 L6 A Library of Parent Functions

Name: _____ HR: _____

Skills: Write linear functions, recognize and graph the eight parent functions, evaluate the greatest integer function, and graph piecewise-defined functions.

Warm-Up: Prerequisite Skills

You will use these three skills all lesson — make sure they are solid.

1. Slope from two points

Ex: $(2,5)$ and $(6,-3)$ $m = \frac{-3-5}{6-2} = \frac{-8}{4} = \frac{-2}{1}$

You try: $(-1,4)$ and $(3,12)$ $m = \frac{8}{4} = \frac{2}{1}$

2. Interval notation

Ex: $x \geq -2$ $[-2, \infty)$

Ex: all real numbers $(-\infty, \infty)$

You try: $x < 3$ → $(-\infty, 3)$

3. Even / Odd (from 1.5)

Ex: $f(x) = x^2 + 1$ $f(-x) = (-x)^2 + 1 = x^2 + 1$ $f(x) = f(-x)$ Even

You try: $f(x) = x^3$ $f(-x) = (-x)^3 = -x^3$ $f(-x) = -f(x)$ ODD

Linear Functions $f(x) = mx + b$ $y = mx + b$

Graph: a line with slope m and y-intercept $(0,b)$. ← $(0,b)$

- Domain: all real numbers $(-\infty, \infty)$
- Range (when $m \neq 0$): $(-\infty, \infty)$
- Increasing when $m > 0$; decreasing when $m < 0$; constant when $m = 0$

$m > 0$ pos $m < 0$ Neg

Two special linear functions

Constant: $f(x) = c$ — graph is a Horizontal line; range is the single number c. $y = c$ constant

$f(x) = c$ $y = c$ $y = 4$

Identity: $f(x) = x$ — slope 1, passes through (0, 0); each output = its input. $y = x$ $(0,0)$ $(1,1)$

Ex: Write the linear function f for which f(1) = 3 and f(4) = 0.

Points: (1, 3) and (4, 0) (x_1, y_1)

Slope: $m = \frac{0-3}{4-1} = \frac{-3}{3} = -1$

Point-slope: $y - y_1 = m(x - x_1)$

$y - 0 = -1(x - 4)$

$f(x) =$ $y = -x + 4$

$f(x) = -x + 4$

You try: Write the linear function f for which f(−2) = 6 and f(4) = −9.

$(-2, 6)$ $(4, -9)$

$m = \frac{-9-6}{4-(-2)} = \frac{-15}{6} = -\frac{5}{2}$

$y - 6 = -\frac{5}{2}(x - (-2))$

$y - 6 = -\frac{5}{2}(x + 2)$

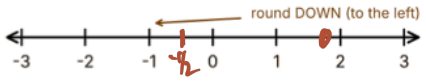
$f(x) =$ $y - 6 = -\frac{5}{2}x - 5$ $+6$ $+6$

$y = -\frac{5}{2}x + 1$

Step Functions: The Greatest Integer Function

[[x]] = the greatest integer less than or equal to x. On a number line, move to the integer at or to the LEFT of x.

Evaluate (round DOWN):



[[−1]] = −1 [[−½]] = −1

[[0.1]] = 0 [[1.5]] = 1

[[1.9]] = 1 [[−2.3]] = −3

Ex: f(x) = [[x]] + 1. Evaluate:

(a) $f(-1) = [[-1]] + 1 = -1 + 1 = 0$

(b) $f(2) = [[2]] + 1 = 2 + 1 = 3$

(c) $f\left(\frac{3}{2}\right) = [[\frac{3}{2}]] + 1 = 1 + 1 = 2$ 1.5

You try: f(x) = [[x + 2]]. Evaluate:

(a) $f(-1.5) = [[-1.5 + 2]] = [[.5]] = 0$

(b) $f(1) = [[1 + 2]] = [[3]] = 3$

(c) $f\left(-\frac{5}{2}\right) = [[-2.5 + 2]] = [[-.5]] = -1$ $f(-2.5)$

Piecewise-Defined Functions

Graph each piece only on its own interval. ≤ or ≥ → solid dot ● < or > → open dot ○

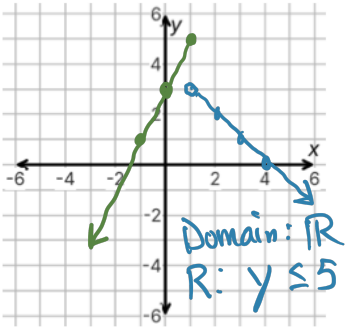
Ex: Sketch the graph of

$$f(x) = \begin{cases} 2x + 3, & x \leq 1 \\ -x + 4, & x > 1 \end{cases}$$

Check the break at x = 1:

Left piece: $f(1) = 2(1) + 3 = 5$ $f(0) = 3$

Right piece: $f(1) = -1 + 4 = 3$ $f(4) = -4 + 4 = 0$



Domain: $\mathbb{R}$ R: $y \leq 5$

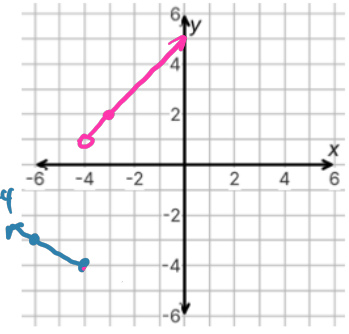
You try: Sketch the graph of

$$f(x) = \begin{cases} -\frac{1}{2}x - 6, & x \leq -4 \\ x + 5, & x > -4 \end{cases}$$

Check the break at x = −4:

Left piece: $f(-4) = -\frac{1}{2}(-4) - 6 = 2 - 6 = -4$ $f(-6) = -\frac{1}{2}(-6) - 6 = 3 - 6 = -3$

Right piece: $f(-4) = 1$ $f(-3) = 2$

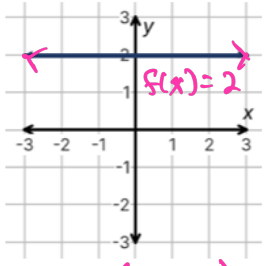


The Library of Parent Functions

Memorize these eight shapes — in Section 1.7 we transform every one of them. Fill in the domain and range, then circle the symmetry.

(a) Constant

$f(x) = c$ (c = 2 shown)



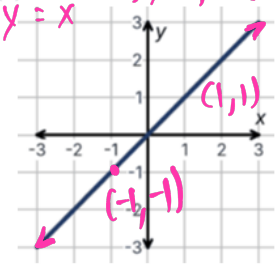
D: $\mathbb{R}$ or $(-\infty, \infty)$

R: $y = 2$

Even Odd Neither

(b) Identity — Parent Linear

$f(x) = x$ (1,1) (3,3) (4,4) $y = x$



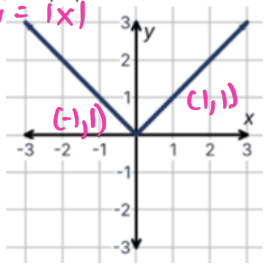
D: $\mathbb{R}$

R: $\mathbb{R}$

Even Odd Neither

(c) Absolute Value

$f(x) = |x|$ $y = |x|$



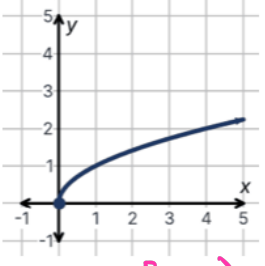
D: $\mathbb{R}$

R: $y \geq 0$ or $[0, \infty)$

Even Odd Neither

(d) Square Root — Radical — Square Root Radical

$f(x) = \sqrt{x}$



D: $x \geq 0$ $[0, \infty)$

R: $y \geq 0$ $[0, \infty)$

Even Odd Neither

(e) Squaring = QUADRATIC — parabola

$f(x) = x^2$



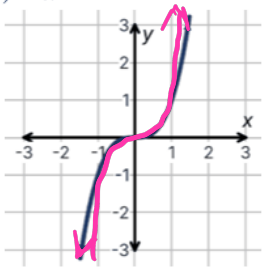
D: $\mathbb{R}$

R: $y \geq 0$

Even Odd Neither

(f) Cubic

$f(x) = x^3$



D: $\mathbb{R}$

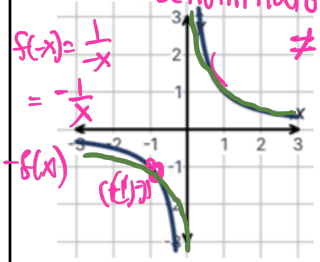
R: $\mathbb{R}$

Even Odd Neither

(g) Reciprocal — Rational — Parent

$f(x) = \frac{1}{x}$ → Denominator $\neq 0$

$f(-x) = \frac{1}{-x} = -\frac{1}{x}$ $-f(x)$



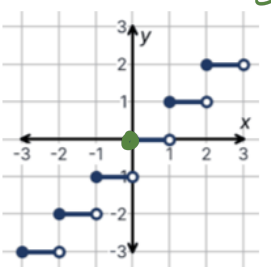
D: $\mathbb{R}, x \neq 0$; $(-\infty, 0) \cup (0, \infty)$

R: $\mathbb{R}, y \neq 0$; $(-\infty, 0) \cup (0, \infty)$

Even Odd Neither

(h) Greatest Integer

$f(x) = [[x]]$ — step functions



$[[x]]$

x	y
0	0
.3	0

D: $\mathbb{R}$

R: Integers: $\{\ldots -2, -1, 0, 1, 2\}$

Even Odd Neither

Quick Check: Name the Parent Function

Each function below is built from one of the eight parents. Write the parent's name — this is exactly what we do first in Section 1.7.

1. $g(x) = x^2 + 3$ — Quad	2. $g(x) = (x-1)^3 + 2$ — Cubic	3. $g(x) = \sqrt{x+4}$ — Square Root
4. $g(x) = \lvert x - 1 \rvert$ — Abs. Value	5. $g(x) = \frac{1}{x-2}$ — Reciprocal / Rational	6. $g(x) = [[x]] - 1$ — Greatest Int. function

Section 1.6 Summary — Can you...

- ☐ Write a linear function from two function values?
- ☐ Sketch and name all eight parent functions from memory?
- ☐ State the domain and range of each parent function?
- ☐ Evaluate the greatest integer function [[x]]?
- ☐ Graph a piecewise function with correct open/closed dots?

Formulas at a Glance — Section 1.6

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Point-slope: $y - y_1 = m(x - x_1)$

$[[x]]$= greatest integer $\leq x$

Solid dot: ≤ or ≥ Open dot: < or >

Even: $f(-x) = f(x)$ Odd: $f(-x) = -f(x)$