U1 L7A Transformations: Shifts & Reflections

Name: _____ HR: _____

Skills: Read function notation to identify shifts (h, k) and reflections, sketch transformed parent functions, and write equations from graphs.

Warm-Up: Prerequisite Skills

1. Key points of a parent	2. Substituting an expression	3. Evaluating with function notation
Ex: $f(x)=\sqrt{x}$ → (0, 0), (1, 1), (4, 2) ($x \geq 0$)	Ex: $f(x)=x^2$ find $f(x-3)$ = $f(x-3)=(x-3)^2=(x-3)(x-3)$ Expand: $x^2-3x-3x+9$ x^2-6x+9	Ex: $f(x)=\lvert x\rvert$ find $f(-2)+1$ $f(-2)+1=\lvert -2\rvert+1=3$
You try: $f(x)=\lvert x\rvert$ → three points: (1, 1) (0, 0) (−1, 1)	You try: $f(x)=x^3$, $f(x+2)$ = $f(x+2)=(x+2)^3$	You try: $f(x)=x^2$, $-f(3)$ = $-f(3)=-(3)^2=-9$

The Big Idea: Inside vs. Outside

The Transformation Template: $g(x)=a\cdot f\big(b(x-h)\big)+k$ (vertical shift)

- a and b are only 1 or −1 (reflections in this section)
- h and k shift.

Note: the textbook uses the letter c for every shift. We use h (inside) and k (outside) so each position has its own letter.

OUTSIDE the parentheses → VERTICAL (a and k)	INSIDE the parentheses → HORIZONTAL (b and h)
Changes the **y-values** (outputs).	Changes the **x-values** (inputs).
Does what it says: $+k$ moves UP, $-k$ moves DOWN.	**Does the OPPOSITE:** $x-h$ moves RIGHT, $x+h$ moves LEFT.
$a=-1$ (a negative in front) flips over the _____-axis.	$b=-1$ ($-x$ inside) flips over the _____-axis.

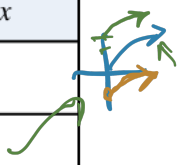
Notation Key — Rigid Transformations

Rigid = the shape does not change, only the position. Fill in the middle columns.

Reflections, Translations

Notation ($h,\ k>0$)	Letter / In or Out	What happens to the graph of f	Example if $f(x)=\sqrt{x}$
$g(x)=f(x)+k$	outside Function K	+K Translates parent up by K	$\sqrt{x}+2$
$g(x)=f(x)-k$	"	−K Translates Parent down by K	$\sqrt{x}-2$
$g(x)=f(x-h)$	inside	x−h → Translates Right by h	$\sqrt{x-2}$
$g(x)=f(x+h)$	inside	x+h → h is neg. : Left by h	$\sqrt{x+2}$
$g(x)=-f(x)$ (a)	outside	x-AXIS	$-\sqrt{x}$
$g(x)=f(-x)$	Inside	y-AXIS	$\sqrt{-x}$





Sign Watch: In $f(x-h)$, h is the number being **subtracted**. $f(x-3)\rightarrow h=3$, moves right 3. $f(x+3)=f(x-(-3))\rightarrow h=-3$, moves left 3.

Practice Reading the Notation —> *No formula needed — just read the left side.*

Ex: Find h and k (and any reflection). Describe how the graph of g compares to the graph of f.

(a) $g(x)=f(x)-5$ Translation down 5 (−K)

(b) $g(x)=f(x+4)$ Translation left 4 (x+h)

(c) $g(x)=f(x-2)+3$ Right 2 up 3

(d) $g(x)=-f(x+3)$ Reflect over x-AXIS, Left 3

① Do Reflections 1st Then translation

You try: Find h and k (and any reflection). Describe how the graph of g compares to the graph of f.

(a) $g(x)=f(x)+7$ → up 7

(b) $g(x)=f(x-6)$ Right 6 (x−h)

(c) $g(x)=f(x+1)-4$ Left 1, Down 4

(d) $g(x)=f(-x)-1$ Reflect over y-AXIS, Down 1

EXAMPLE ACTIVITY (4) Examples a & b

One skill. Your own practice.

EXAMPLE ACTIVITY (5) Examples C & D

From Notation to Graph

Step 1: name the parent. Step 2: write it in f-notation. Step 3: find h and k. Step 4: sketch.

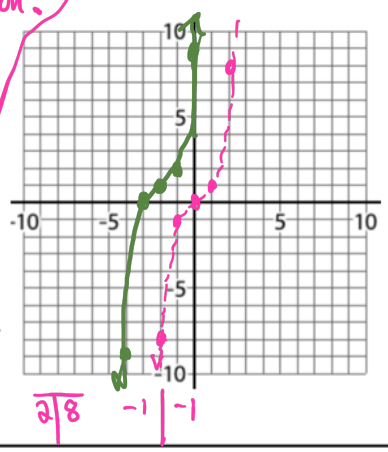
Ex: Use the graph of $f(x) = x^3$ ***to sketch*** $g(x) = (x+2)^3 + 1$.

Parent: Cubic function.

In f-notation: $g(x) =$ $g(x) = f(x+2) + 1$

h = −2 k = 1

Transformations: left 2 & up 1



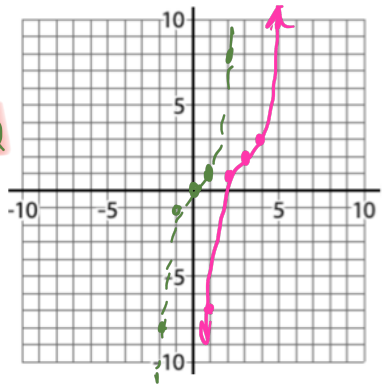
You try: Use the graph of $f(x) = x^3$ ***to sketch*** $g(x) = (x-3)^3 + 2$.

Parent: Cubic, x^3

In f-notation: $g(x) =$ $g(x) = f(x-3) + 2$

h = 3 k = 2

Transformations: Right 3 up 2



Reflections in the Coordinate Axes: $-f(x)$ $(a = -1)$ *flips over the x-axis.* $f(-x)$ $(b = -1)$ *flips over the y-axis.*

Discuss

Function	f-notation	Compare to the graph of $f(x) = \sqrt{x}$	Domain
Ex: $g(x) = -\sqrt{x}$ (↑ outside)	$g(x) =$ $-f(x)$	Reflect over x-axis	(x can't be neg.) $x \geq 0$
Ex: $h(x) = \sqrt{-x}$ (↑ inside)	$h(x) =$ $f(-x)$	Reflected over y-axis	$-x \geq 0$; $x \leq 0$
Ex: $k(x) = -\sqrt{x+2}$	$k(x) =$ $-f(x+2)$	Reflect over x-axis Then left 2	$x+2 \geq 0$; $x \geq -2$

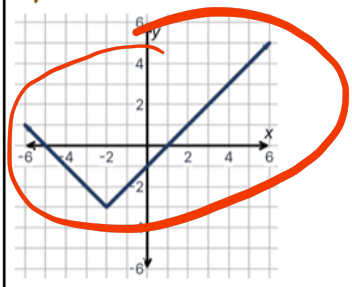
You try: Now let $f(x) = \sqrt{x-1}$. Compare the graphs of $g(x) = -\sqrt{x-1}$ and $h(x) = \sqrt{-x-1}$ to the graph of f.

g: $g(x) = -f(x)$ $f(x) = \sqrt{x-1}$ $g(x) = -\sqrt{x-1}$

h: $h(x) = f(-x)$ $h(x) = \sqrt{-x-1}$ $= \sqrt{-1(x+1)}$

Writing Equations from Graphs: *Find the parent, then find where the anchor point (vertex or starting point) moved: that point is* (h, k).

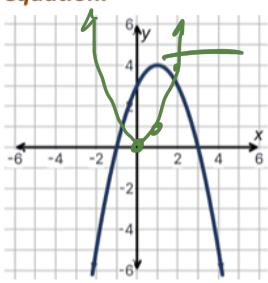
Ex: The graph is a transformation of $f(x) = |x|$***. Write its equation.***



h = −2 k = −3 $g(x) =$ $f(x+2) - 3$ ← in terms of f

$= |x+2| - 3$

You try: The graph is a transformation of $f(x) = x^2$***. Write its equation.***



h = 2 k = 4 $g(x) =$ $-f(x-2) + 4$

$= -(x-2)^2 + 4$

Skill 6

Section 1.7A Summary — Can you...

- ☐ Read the left side and name h, k, and any reflection?
- ☐ Rewrite a function like $(x+2)^3 + 1$ in f-notation?
- ☐ Sketch a shifted or reflected parent and write an equation from a graph?

Order Tip

When a graph is reflected AND shifted, apply the reflection first, then the shifts.

Example: $-|x-3| + 2$ → flip $|x|$ over the x-axis, then move right 3 and up 2.