U1 L7B Transformations: Stretches & Shrinks — NOTES

Name: _____ HR: _____

Skills: Read function notation to identify vertical and horizontal stretches and shrinks (a and b), map points, and combine transformations with g(x) = a·f(b(x − h)) + k.

Warm-Up: Prerequisite Skills

1. Describe Transformation (1.7A)	2. Substituting an expression	3. Multiplying an output
Ex: $g(x) = f(x+2) - 3 \rightarrow$ left 2 down 3	Ex: $f(x) = x^2$, $f(2x) =$ $(2x)^2$ or $4x^2$	Ex: $f(x) = x^3$, $3f(2) =$ $3(2)^3 = 3(8) = 24$
You try: $g(x) = -f(x-1) + 4 \rightarrow$ Reflected/flipped x-axis, shift R 1, up 4	You try: $f(x) = x^3$, $f\left(\frac{1}{2}x\right) =$ $\left(\frac{1}{2}x\right)^3$ or $\frac{1}{8}x^3$	You try: $f(x) = \lvert x \rvert$, $\frac{1}{2}f(-6) =$ $\frac{1}{2}(\lvert -6 \rvert) = \frac{1}{2}(6) = 3$

The Transformation Template: $g(x) = a \cdot f\big(b(x-h)\big) + k$

OUTSIDE (vertical — does what it says): a stretches or shrinks the y-values (negative a → flip over the x-axis); k moves up or down.

INSIDE (horizontal — does the opposite): b shrinks or stretches the x-values by $\frac{1}{b}$ (negative b → flip over the y-axis); h moves right or left.

Order: stretch/shrink and reflect first, then shift.

Notation Key — Nonrigid Transformations (a and b)

Nonrigid = the shape changes. Same rule as 1.7A: outside → vertical, does what it says; inside → horizontal, does the opposite.

Notation	In or Out	What happens to the graph of f	Point (x, y) becomes
$a \cdot f(x),\ a > 1$	Outside	vertical Stretch by a factor of a (taller)	$(x,\ ay)$
$a \cdot f(x),\ 0 < a < 1$	outside	vertical Shrink by a factor of a (flatter)	$(x,\ ay)$
$f(bx),\ b > 1$ (b=2, b=3)	inside	horizontal Shrink by a factor of $\frac{1}{b}$ (narrower)	$\left(\frac{x}{b},\ y\right)$
$f(bx),\ 0 < b < 1$ (b=.25, b=1/4)	inside	horizontal Stretch by a factor of $\frac{1}{b}$ (wider)	$\left(\frac{x}{b},\ y\right)$

Anchor example: $g(x) = 3f(2x)$ The 3 is OUTSIDE → vertical stretch by 3. The 2 is INSIDE → the ~~opposite~~ reciprocal → horizontal shrink by 1/2. $(x,\ y) \rightarrow \left(\frac{x}{2},\ 3y\right)$

Do Transformation Explorer

Practice Reading the Notation

Find a, b, h, and k, then describe how the graph of g compares to the graph of f. No formula needed.

Ex:

(a) $g(x) = 3f(2x)$ vert. stretch by factor 3, Horiz shrink by 1/2

(b) $g(x) = \frac{1}{2}f(x) - 4$ vert. shrink by 1/2, shift down 4

(c) $g(x) = f\left(\frac{1}{4}x\right)$ b=1/4 Horizontal stretch by factor 4

(d) $g(x) = -2f(x+1)$ ref. over x-Axis, v-stretch by 2, Left 1

You try:

(a) $g(x) = 2f(x+5)$ V. Stretch factor 2, Left 5

(b) $g(x) = f(3x) + 1$ H. Shrink by 1/3, up 1

(c) $g(x) = \frac{1}{3}f(x-2)$ $g(x) = f\left(\frac{1}{5}x\right)$ H. stretch by 5

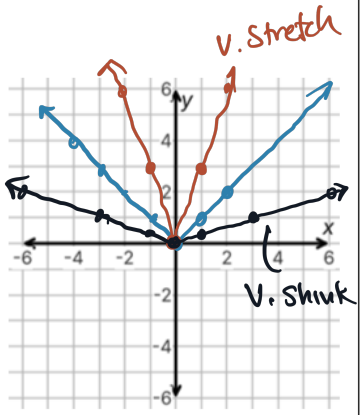
(d) $g(x) = f(-2x)$

Do function transformations

Vertical Stretches & Shrinks with a Table $f(x) = |x|$

Ex: Compare the graphs of $h(x) = 3|x|$ ***and*** $g(x) = \frac{1}{3}|x|$ ***to the graph of*** $f(x) = |x|$.

x	−3	−1	0	1	3
$f(x) = \|x\|$	3	1	0	1	3
	×3	×3	×3		
$h(x) = 3\|x\|$	9	3	0	3	9
$g(x) = \frac{1}{3}\|x\|$	3×1/3 = 1	1×1/3 = 1/3	0	1/3	1



h is a Vertical Stretch of f.

g is a Vertical Shrink of f.



Horizontal Stretches & Shrinks with a Formula

Ex: Let $f(x) = 2 - x^3$. ***Write and describe each function.***

(a) $g(x) = f(2x) =$ $2-(2x)^3$ or $2-8x^3$ ← vertical Stretch

Horizontal shrink by $\frac{1}{2}$

(b) $h(x) = f\left(\frac{1}{2}x\right) =$ $2-\left(\frac{1}{2}x\right)^3$ or $2-\frac{1}{8}x^3$ ← vert shrink

factor: 2 → horizontal stretch by 2

You try: Let $f(x) = x^2 + 3$. ***Write and describe each function.***

(a) $g(x) = f(2x) =$ $(2x)^2+3$

H. Shrink by factor $\frac{1}{2}$

(b) $h(x) = f\left(\frac{1}{2}x\right) =$ $\left(\frac{1}{2}x\right)^2+3$

H. Stretch by 2

Mapping Points

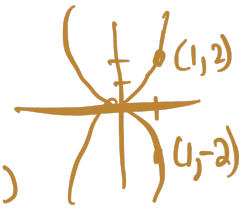
Use the point-mapping rule from the template box on page 1.

Ex: The point (4, −2) is on the graph of f. Find the matching point on g.

(a) $g(x) = 3f(2x)$ (4, −2) → (2, −6)

Horizontal shrink by $\frac{1}{2}$; V. stretch by 3; f(x) → g(x)

(b) $g(x) = -f(x-1) + 3$ (4, −2) → (5, 5)



(5, 2)

You try: The point (4, −2) is on the graph of f. Find the matching point on g.

(a) $g(x) = \frac{1}{2}f(x) + 1$ (4, −2) → (4, 0)

(, −1)

(b) $g(x) = f\left(\frac{1}{2}x\right)$ (4, −2) → (8, −2)

(c) $g(x) = 2f(x+3)$ (4, −2) → (1, −4)

×2 V. Stretch

Putting It All Together

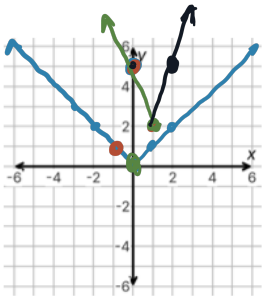
Ex: Sketch $g(x) = 3|x-1| + 2$ ***by mapping key points of*** $f(x) = |x|$. ← $a|x-h|+k$

a = 3 h = 1 k = 2

(−1, 1) → (0, 5) (0, 3)

(0, 0) → (1, 2)

(1, 1) → (2, 5)



You try: Sketch $g(x) = -2|x+1| - 3$ ***by mapping key points of*** $f(x) = |x|$.

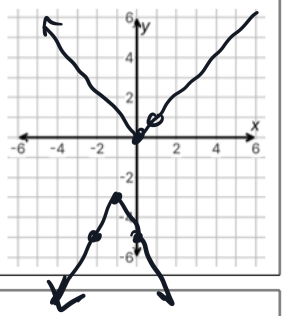
a = −2 h = −1 k = −3

x−2 sub 3

(−1, 1) → (−2, −5)

(0, 0) → (−1, −3)

(1, 1) → (0, −5)



Section 1.7B Summary — Can you...

- ☐ Name a, b, h, and k from the left side?
- ☐ Map a point using the point-mapping rule?
- ☐ Sketch a stretched or shrunk parent function?

Watch Out

- b inside does the OPPOSITE: $f(2x)$ is a horizontal SHRINK by ½.
- Stretch/shrink and reflect first, then shift.