U1 L7B Transformations: Stretches & Shrinks — NOTES

Name: _____ HR: _____

Skills: Read function notation to identify vertical and horizontal stretches and shrinks (a and b), map points, and combine transformations with g(x) = a·f(b(x − h)) + k.

Warm-Up: Prerequisite Skills

1. Describe Transformation (1.7A)	*2. Substituting an expression*	*3. Multiplying an output*
Ex: $g(x) = f(x+2) - 3 \rightarrow$	Ex: $f(x) = x^2,\ f(2x) =$	Ex: $f(x) = x^3,\ 3f(2) =$
You try: $g(x) = -f(x-1) + 4 \rightarrow$	You try: $f(x) = x^3,\ f\left(\frac{1}{2}x\right) =$	You try: $f(x) = \lvert x \rvert,\ \frac{1}{2}f(-6) =$

The Transformation Template: $g(x) = a \cdot f\left(b(x-h)\right) + k$

OUTSIDE (vertical — does what it says): a stretches or shrinks the y-values (negative a → flip over the x-axis); k moves up or down.

INSIDE (horizontal — does the opposite): b shrinks or stretches the x-values by $\frac{1}{b}$ (negative b → flip over the y-axis); h moves right or left.

Order: stretch/shrink and reflect first, then shift.

Notation Key — Nonrigid Transformations (a and b)

Nonrigid = the shape changes. Same rule as 1.7A: outside → vertical, does what it says; inside → horizontal, does the opposite.

Notation	In or Out	What happens to the graph of f	Point (x, y) becomes
$a \cdot f(x),\ a > 1$		vertical _____ by a factor of a (taller)	$(x,\ ay)$
$a \cdot f(x),\ 0 < a < 1$		vertical _____ by a factor of a (flatter)	$(x,\ ay)$
$f(bx),\ b > 1$		horizontal _____ by a factor of $\frac{1}{b}$ (narrower)	$\left(\frac{x}{b},\ y\right)$
$f(bx),\ 0 < b < 1$		horizontal _____ by a factor of $\frac{1}{b}$ (wider)	$\left(\frac{x}{b},\ y\right)$

Anchor example: $g(x) = 3f(2x)$ The 3 is OUTSIDE → vertical stretch by ____. The 2 is INSIDE → the opposite → horizontal shrink by _____. $(x,\ y) \rightarrow \left(\frac{x}{2},\ 3y\right)$

Practice Reading the Notation

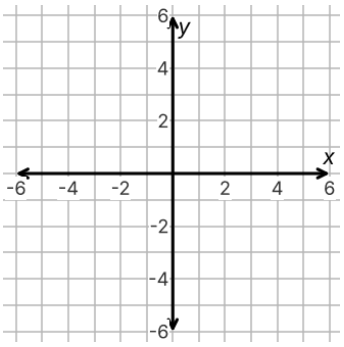
Find a, b, h, and k, then describe how the graph of g compares to the graph of f. No formula needed.

Ex:	*You try:*
(a) $g(x) = 3f(2x)$	(a) $g(x) = 2f(x+5)$
(b) $g(x) = \frac{1}{2}f(x) - 4$	(b) $g(x) = f(3x) + 1$
(c) $g(x) = f\left(\frac{1}{4}x\right)$	(c) $g(x) = \frac{1}{3}f(x-2)$
(d) $g(x) = -2f(x+1)$	(d) $g(x) = f(-2x)$

Vertical Stretches & Shrinks with a Table

Ex: Compare the graphs of $h(x) = 3|x|$ ***and*** $g(x) = \frac{1}{3}|x|$ ***to the graph of*** $f(x) = |x|$.

x	−3	−1	0	1	3
$f(x) = \|x\|$	3	1	0	1	3
$h(x) = 3\|x\|$					
$g(x) = \frac{1}{3}\|x\|$					



h is a _____ of f.

g is a _____ of f.

Horizontal Stretches & Shrinks with a Formula

Ex: Let $f(x) = 2 - x^3$. ***Write and describe each function.***

(a) $g(x) = f(2x)$ =

(b) $h(x) = f\left(\frac{1}{2}x\right)$ =

You try: Let $f(x) = x^2 + 3$. ***Write and describe each function.***

(a) $g(x) = f(2x)$ =

(b) $h(x) = f\left(\frac{1}{2}x\right)$ =

Mapping Points

Use the point-mapping rule from the template box on page 1.

Ex: The point (4, −2) is on the graph of f. Find the matching point on g.

(a) $g(x) = 3f(2x)$

(b) $g(x) = -f(x-1) + 3$

You try: The point (4, −2) is on the graph of f. Find the matching point on g.

(a) $g(x) = \frac{1}{2}f(x) + 1$

(b) $g(x) = f\left(\frac{1}{2}x\right)$

(c) $g(x) = 2f(x+3)$

Putting It All Together

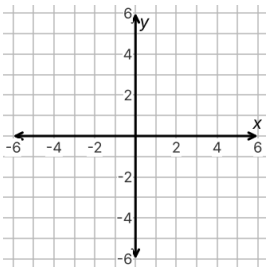
Ex: Sketch $g(x) = 3|x-1| + 2$ ***by mapping key points of*** $f(x) = |x|$.

a = h = k =

(−1, 1) →

(0, 0) →

(1, 1) →



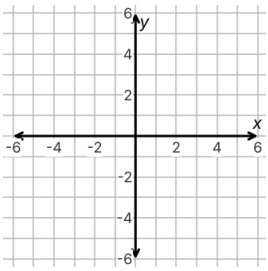
You try: Sketch $g(x) = -2|x+1| - 3$ ***by mapping key points of*** $f(x) = |x|$.

a = h = k =

(−1, 1) →

(0, 0) →

(1, 1) →



Section 1.7B Summary — Can you…

☐ Name a, b, h, and k from the left side?

☐ Map a point using the point-mapping rule?

☐ Sketch a stretched or shrunk parent function?

Watch Out

• b inside does the OPPOSITE: $f(2x)$ is a horizontal SHRINK by ½.

• Stretch/shrink and reflect first, then shift.